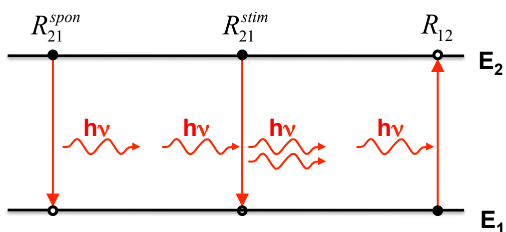
EE 232 Lightwave Devices
Lecture 8: Einstein's AB Coefficients, Spontaneous Emission

Instructor: Ming C. Wu

University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

Einstein's AB Coefficients

$$R_{21}^{spon} = A_{21} f_2 \left(1 - f_1\right)$$
$$R_{21}^{stim} = B_{21} f_2 \left(1 - f_1\right) P(E_{21})$$
$$R_{12} = B_{12} f_1 \left(1 - f_2\right) P(E_{21})$$



For non-monochromatic light:

$P(E_{21}) = n_{ph} N(E_{21})$:

number of photons per unit volume per energy interval

$$n_{ph} = \frac{1}{e^{\hbar\omega_k / k_B T} - 1}$$: Number of photons per state (Bose-Einstein distribution)

$$N\left(E_{21}\right) = \frac{8\pi n_r^3 E_{21}^2}{h^3 c^3}$$:Number of states with photon energy E_{ba} per unit volume, per energy interval

Photon Density of States

Optical wave $e^{i\vec{k}\cdot\vec{r}}$ satisfies periodic boundary condition

$\omega_k = \dfrac{kc}{n_r}$ dispersion relation of photons

(equivalent to energy band structure of electrons)

Number of states with photon energy E_{21} per unit volume, per energy interval

$$N\left(E_{21}\right) = \frac{2}{V}\int \frac{4\pi k^2 dk}{\left(\frac{2\pi}{L}\right)^3}\cdot\delta(E_{21} - \hbar\omega_k)$$

$$= \frac{8\pi}{\left(2\pi\right)^3}\int\left(\frac{n_r\omega_k}{c}\right)^2\frac{n_r}{c}d\omega_k\cdot\frac{1}{\hbar}\delta(\frac{E_{21}}{\hbar} - \omega_k)$$

$$\boxed{N\left(E_{21}\right) = \frac{8\pi n_r^3 E_{21}^2}{h^3c^3}}$$

Einstein's AB Coefficients

At thermal equilibrium:

$$R_{12} = R_{21}^{spon} + R_{21}^{stim}$$

$$B_{12}f_1\left(1-f_2\right)P(E_{21}) = A_{21}f_2\left(1-f_1\right) + B_{21}f_2\left(1-f_1\right)P(E_{21})$$

$$P(E_{21}) = \frac{A_{21}f_2\left(1-f_1\right)}{B_{12}f_1\left(1-f_2\right) - B_{21}f_2\left(1-f_1\right)} = \frac{A_{21}e^{\frac{E_1-F}{k_BT}}}{B_{12}e^{\frac{E_2-F}{k_BT}} - B_{21}e^{\frac{E_1-F}{k_BT}}}$$

$$N(E_{21})\cdot n_{ph} = \frac{A_{21}}{B_{12}e^{\frac{E_2-E_1}{k_BT}} - B_{21}} \quad\Rightarrow\quad \left(\frac{8\pi n_r^3E_{21}^2}{h^3c^3}\right)\frac{1}{e^{\hbar\omega_k/k_BT} - 1} = \frac{A_{21}}{B_{12}e^{\frac{E_2-E_1}{k_BT}} - B_{21}}$$

$$\boxed{\begin{array}{l} B_{12} = B_{21} \\ \dfrac{A_{21}}{B_{21}} = \dfrac{8\pi n_r^3E_{21}^2}{h^3c^3} = N(E_{21}) \end{array}}$$

Spontaneous Emission Spectra

$$B_{12} = B_{21} = B$$

$$\frac{A_{21}}{B} = \frac{8\pi n_r^3 E_{21}^2}{h^3 c^3} = N(E_{21})$$

$$R_{21}^{spon} = r_{21}^{spon}(E_{21})dE = A_{21} f_2 (1 - f_1)$$

$$R_{net}^{abs} = r_{net}^{abs}(E_{21})dE = B\left[f_2 - f_1\right]P(E_{21})$$

Absorption coefficient:

$$\alpha(E_{21})dE = \frac{r_{net}^{abs}(E_{21})dE}{P(E_{21})(c/n_r)} = \frac{n_r}{c} B\left[f_1 - f_2\right] = -g(E_{21})dE$$

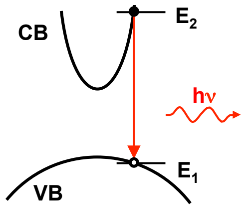
$$\frac{r_{21}^{spon}(E_{21})}{g(E_{21})} = \frac{A_{21}}{\frac{n_r}{c} B}\left[\frac{f_2(1-f_1)}{f_2 - f_1}\right]$$

n_{sp}

$$r_{21}^{spon}(E_{21}) = \frac{8\pi n_r^2 E_{21}^2}{h^3 c^2}\left[\frac{1}{1 - e^{\frac{E_{21} - \Delta F}{k_B T}}}\right] g(E_{21}) \qquad [\frac{1}{s}\frac{1}{m^3}\frac{1}{eV}]$$

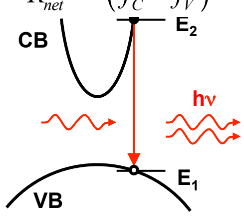
Spontaneous Emission

$$R^{spon} \propto f_C\left(1 - f_V\right)$$

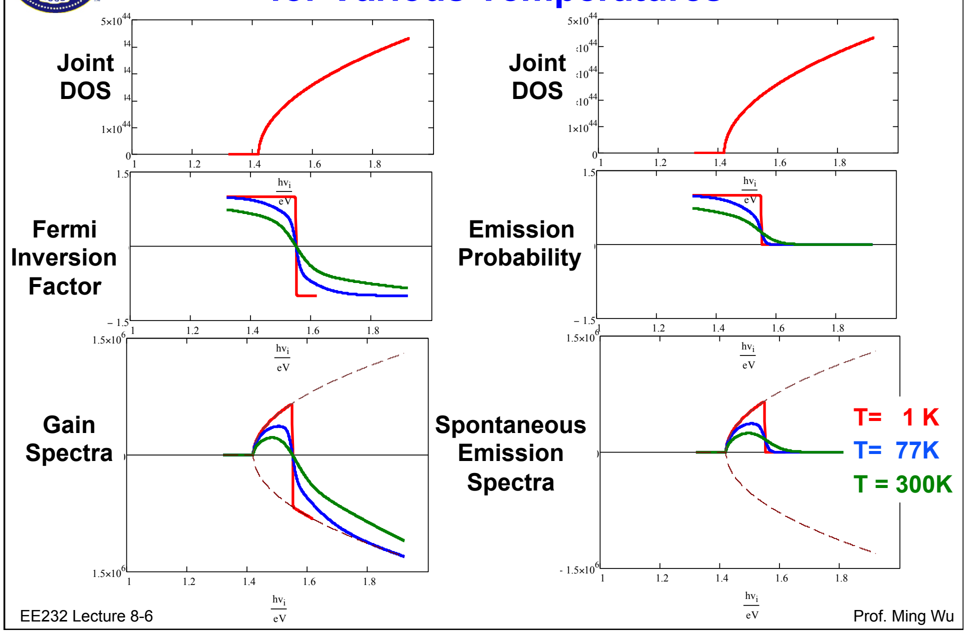


Stimulated Emission

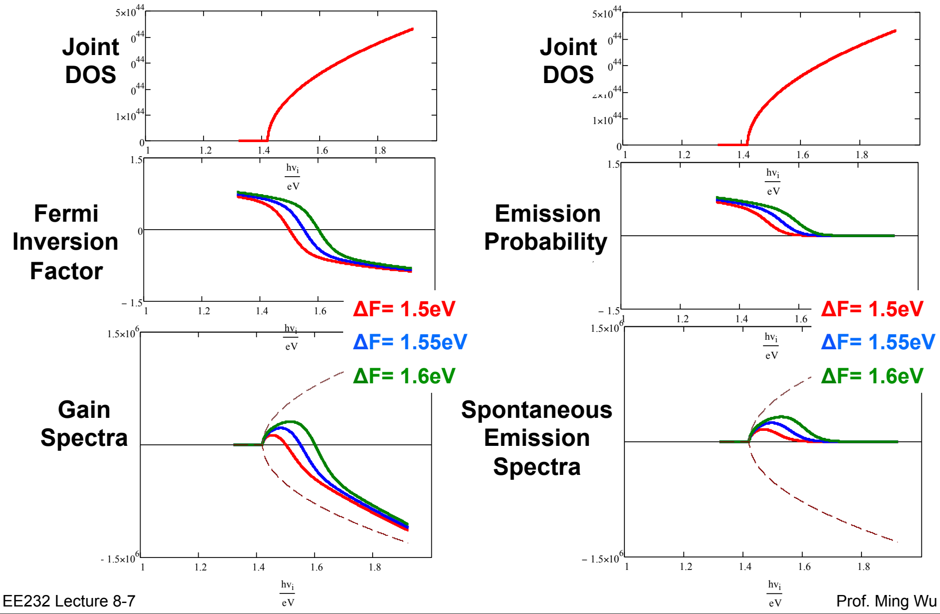
$$R_{net}^{stim} \propto \left(f_C - f_V\right)$$



Spontaneous Emission and Gain Spectra for Various Temperatures



Spontaneous Emission and Gain Spectra for ΔF (T = 300 K)



Spontaneous Emission Lifetime

$$r_{21}^{spon}(\hbar\omega) = \frac{1}{\tau_r}\rho_r(\hbar\omega - E_g) f_e(\hbar\omega)$$

$$f_e(\hbar\omega) = f_C(E_2)\left(1 - f_V(E_1)\right)$$

$$r_{21}^{spon}(E_{21}) = \frac{8\pi n_r^2 E_{21}^2}{h^3 c^2} \frac{1}{1 - e^{\frac{E_{21}-\Delta F}{k_B T}}} g(E_{21})$$

$$= \frac{8\pi n_r^2 E_{21}^2}{h^3 c^2} \frac{f_e(\hbar\omega)}{f_g(\hbar\omega)} \left(C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r(\hbar\omega - E_g) \right) f_g(\hbar\omega)$$

$$\Rightarrow \tau_r = \frac{h^3 c^2}{8\pi n_r^2 E_{21}^2} \cdot \frac{1}{C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2}$$

Typically $\tau_r \sim 1$ nsec

$$\text{Flux\_per\_eV} := \frac{8\cdot\pi\cdot nr^2\cdot Eg^2}{(2\cdot\pi\cdot h\_bar)^3\cdot c^2} \qquad \text{Flux\_per\_eV} = 6.078\times 10^{47}\ \frac{s}{m^4\cdot kg}$$

$$\tau\_r := \frac{1}{\text{Flux\_per\_eV}}\cdot\frac{1}{C0\cdot\frac{m0}{6}\cdot Ep}$$

$$\tau\_r = 5.443\times 10^{-10}\ s$$

Alternative Form for Gain

$$C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2=\frac{h^3c^2}{8\pi n_r^2E_{21}^2}\cdot\frac{1}{\tau_r}=\frac{h\lambda_0^2}{8\pi n_r^2\tau_r}$$

$$g(\hbar\omega)=\frac{h\lambda_0^2}{8\pi n_r^2\tau_r}\rho_r(\hbar\omega-E_g)f_g(\hbar\omega)$$